

On the existence of natural self-oscillation of a free electron

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Abstract

The possibility of the existence of natural self-oscillation of a free electron is suggested. This oscillation depends on the interaction of the electron with its own electromagnetic fields. Suitable standing wave solutions of the electromagnetic fields are chosen. A kind of displacement dependent electric potential and mechanism of energy exchange between velocity and acceleration dependent electromagnetic fields are analyzed. Conditions for the existence of natural self-oscillation are given.

Keywords:

natural self-oscillation, standing wave solution, displacement dependent potential

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1. Introduction

The subtle and beautiful experiment of Dehmelt et al showed that a single isolated electron might be driven by static electric field in a Penning trap to oscillate as a “mono-electron oscillator”[1]. We want to show here that a free electron may also possess natural self-oscillation by itself under some selected conditions.

The property of a free electron has been discussed by many authors through both classical and quantum theories[2, 3]. There are still difficulties

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and paradoxes about the model of electron. Here we take a simple model of electron used by Konopinsky[4] and Dehmelt[5], in which a free electron is a system consisting of a particle with mass m , charge e and distributed electromagnetic fields. These fields may have their own energies, momentums and angular momentums. The more detailed internal structures of the electron are neglected at first, thus many difficulties about the model of electron are avoided.

2. Standing wave solutions of electromagnetic fields of an electron in harmonic oscillation

The electromagnetic fields of an electron in harmonic oscillation may be derived from the vector potential or Hertz vector of the oscillating electron[6, 7, 8]. Following the method of Bateman[9] or Adler, Chu and Fano[10], we may also solve the wave equation of the electromagnetic fields on the basis of Maxwell equations in spherical coordinates first, and then match the solutions with the motion of the electron. From both methods we may have the dipole term of the electron's electromagnetic fields as[6]

$$\mathbf{E}_r = 2eZ_{e0} \left[\frac{1}{r^3} - \frac{ik}{r^2} \right] \cos \theta e^{i\omega t} e^{\mp ikr} \hat{\mathbf{r}} \quad (1)$$

$$\mathbf{E}_\theta = eZ_{e0} \left[\frac{1}{r^3} - \frac{ik}{r^2} - \frac{k^2}{r} \right] \sin \theta e^{i\omega t} e^{\mp ikr} \hat{\theta} \quad (2)$$

$$\mathbf{H}_\phi = eZ_{e0} \left[-\frac{ik}{r^2} - \frac{k^2}{r} \right] \sin \theta e^{i\omega t} e^{\mp ikr} \hat{\phi} \quad (3)$$

where \mathbf{E}_r and \mathbf{E}_θ are electric fields in r and θ directions, \mathbf{H}_ϕ is magnetic field in ϕ direction, $\hat{\mathbf{r}}$, $\hat{\theta}$ and $\hat{\phi}$ are corresponding unit vectors (Besides (1), (2) and (3), there are also a monopole Coulomb field of the electron and a dipole field at the origin[7], but they are not useful in following analysis), r is the distance from the electron to a field point, e is charge of electron, Z_{e0} is amplitude of its oscillation, terms of $1/r^3$, $1/r^2$ and $1/r$ correspond to oscillating static fields, velocity dependent near fields and acceleration dependent fields respectively.

The \mp signs in the space phase factor represent that retarded and advanced time solutions are used respectively. These two are complex conjugate solutions which represent progressive divergent wave and regressive convergent wave, and any combination of them is also an allowed solution. Usually

the solution of advanced time is rejected by the requirement of causality, but here it is used as one of the conjugate waves, there is no problem about causality. According to Adler, Chu and Fano the two kinds of space factors may add together to form complete standing wave modes[10] of $\sin kr$ and $\cos kr$. We choose the sum of retarded time and advanced time solutions to form standing wave solutions. Then their space and time phase factors are formed as

$$e^{i(\omega t-kr)} + e^{i(\omega t+kr)} = 2(\cos kr \cos \omega t + i \cos kr \sin \omega t) \quad (4)$$

thus for the static and acceleration dependent fields, the phase factors are $2 \cos kr \cos \omega t$. For the velocity dependent electromagnetic fields, they are $2 \cos kr \sin \omega t$, since

$$-i2(\cos kr \cos \omega t + i \cos kr \sin \omega t) = 2(\cos kr \sin \omega t - i \cos kr \cos \omega t) \quad (5)$$

In order to match the motion of the oscillating electron, we have to choose the static electric fields E_{sr} , $E_{s\theta}$ and acceleration dependent electromagnetic fields $E_{a\theta}$, $H_{a\phi}$ with their phase factors as

$$E_{sr} = \frac{4eZ_{eo}}{r^3} \cos \theta \cos kr \cos \omega t \quad E_{s\theta} = \frac{2eZ_{eo}}{r^3} \sin \theta \cos kr \cos \omega t \quad (6)$$

$$E_{a\theta} = \frac{2k^2 e Z_{eo}}{r} \sin \theta \cos kr \cos \omega t \quad H_{a\phi} = \frac{2k^2 e Z_{eo}}{r} \sin \theta \cos kr \cos \omega t \quad (7)$$

The velocity dependent electromagnetic fields E_{vr} , $E_{v\theta}$ and $H_{v\phi}$ with their standing wave phase factors are chosen as

$$E_{vr} = \frac{4keZ_{eo}}{r^2} \cos \theta \cos kr \sin \omega t \quad E_{v\theta} = \frac{2keZ_{eo}}{r^2} \sin \theta \cos kr \sin \omega t \quad (8)$$

$$H_{v\phi} = \frac{2keZ_{eo}}{r^2} \sin \theta \cos kr \sin \omega t \quad (9)$$

3. Displacement dependent electric potential and force for self-oscillation of a free electron

It is known that for any electric field E at a point in free space, there is a corresponding energy density \mathcal{E}_d , which is

$$\mathcal{E}_d = \frac{1}{8\pi} E^2 \quad (10)$$

and the total energy \mathcal{E} of the electric field is the volume integration of its energy density through whole space, which is

$$\mathcal{E} = \frac{1}{8\pi} \int E^2 dv \quad (11)$$

Thus for the electrical fields E_{sr} and $E_{s\theta}$ of the static zone, there is the corresponding energy \mathcal{E}_s . As the time factor is suppressed at first,

$$\begin{aligned} \mathcal{E}_s &= \frac{1}{8\pi} \int (E_{sr}^2 + E_{s\theta}^2) dv \\ &= \frac{1}{8\pi} \int_{r_{min}}^{r_{max}} \left[\left(\frac{4eZ_{eo}}{r^3} \cos \theta \cos kr \right)^2 + \left(\frac{2eZ_{eo}}{r^3} \sin \theta \cos kr \right)^2 \right] 2\pi r^2 dr \sin \theta d\theta \end{aligned} \quad (12)$$

where r_{max} and r_{min} are upper and lower limits of the integration. It's reasonable to take the upper limit r_{max} as infinite or the total length of certain number of standing wave lengths, and the lower limit r_{min} as $M_s r_0$, where r_0 is taken as the classical radius of electron which is defined as[11]

$$r_0 = \frac{e^2}{mc^2} = 2.82 \times 10^{-13} cm \quad (13)$$

and M_s is an undetermined numerical constant. Taking $r_{max} = \infty$ and $r_{min} = M_s r_0$ we get the integration of \mathcal{E}_s as

$$\begin{aligned} \mathcal{E}_s &= 4e^2 Z_{eo}^2 \left[\frac{1 + \cos(2kM_s r_0)}{6(M_s r_0)^3} - \frac{k \sin(2kM_s r_0)}{6(M_s r_0)^2} \right. \\ &\quad \left. - \frac{k^2 \cos(2kM_s r_0)}{3M_s r_0} + \frac{2}{3} k^3 \int_{M_s r_0}^{\infty} \frac{\sin 2kr}{r} dr \right] \end{aligned} \quad (14)$$

Since r_0 and $M_s r_0$ are comparatively small, we may use the approximations

$$\cos(2kM_s r_0) \approx 1 \quad \sin(2kM_s r_0) \approx 2kM_s r_0 \quad \int_{M_s r_0}^{\infty} \frac{\sin 2kr}{r} dr \approx \frac{\pi}{2} \quad (15)$$

Then we have

$$\mathcal{E}_s = 4e^2 Z_{eo}^2 \left[\frac{1}{3(M_s r_0)^3} - \frac{2k^2}{3M_s r_0} + \frac{\pi}{3} k^3 \right] \quad (16)$$

Since Z_{e0} is the amplitude of the harmonic oscillation of the electric fields, \mathcal{E}_s is a displacement dependent energy which is proportional to the square of Z_{e0} and may have a kind of restore force with maximum value f_{s0} , which is

$$f_{s0} = -\frac{\partial \mathcal{E}_s}{\partial Z_{e0}} = -8e^2 \left[\frac{1}{3(M_s r_0)^3} - \frac{2k^2}{3M_s r_0} + \frac{\pi}{3} k^3 \right] Z_{e0} \quad (17)$$

This restore force will drive the electron to take harmonic oscillation and we may take energy \mathcal{E}_s equal the maximum of the electron's kinetic energy \mathcal{E}_k , which is

$$\mathcal{E}_k = \frac{1}{2} m v_0^2 = \frac{1}{2} m c^2 k^2 Z_{e0}^2 \quad (18)$$

since $v_0 = ckZ_{e0}$ is the amplitude of the electron's velocity. Then we have

$$\mathcal{E}_s = \mathcal{E}_k \quad (19)$$

$$4e^2 Z_{e0}^2 \left[\frac{1}{3(M_s r_0)^3} - \frac{2k^2}{3M_s r_0} + \frac{\pi}{3} k^3 \right] = \frac{1}{2} m c^2 k^2 Z_{e0}^2 \quad (20)$$

$$\frac{\pi}{3} k^3 - \left(\frac{2}{3M_s r_0} + \frac{\frac{1}{2} m c^2}{e^2} \right) k^2 + \frac{1}{3(M_s r_0)^3} = 0 \quad (21)$$

then by (13) we get

$$\frac{\pi}{3} k_s^3 - \left(\frac{2}{3M_s r_0} + \frac{1}{8r_0} \right) k_s^2 + \frac{1}{3(M_s r_0)^3} = 0 \quad (22)$$

where k is labeled as k_s . (22) gives the numerical relation of k_s with M_s and r_0 , and is one of the selection conditions for the natural self-oscillation of the free electron. This equation may be modified by some factors such as the range of the upper limit r_{max} , the relativistic variation of the electron's kinetic energy, but the main feature of (22) is not affected.

4. The energy storage and exchange of the velocity and acceleration dependent electromagnetic fields

Using the electromagnetic fields E_{vr} , $E_{v\theta}$ and $H_{v\phi}$ of (8) and (9), we may calculate the energy of the velocity dependent field \mathcal{E}_v as

$$\begin{aligned} \mathcal{E}_v &= \frac{1}{8\pi} \int (E_{vr}^2 + E_{v\theta}^2 + H_{v\phi}^2) dv \\ &= \frac{4k^2 e^2 Z_{e0}^2}{8\pi} \int_{r_{min}}^{r_{max}} \frac{\cos^2 kr}{r^4} (4\cos^2 \theta + 2\sin^2 \theta) 2\pi r^2 dr \sin \theta d\theta \end{aligned} \quad (23)$$

Taking $r_{max} = \infty$ and $r_{min} = M_v r_0$, where M_v is also an undetermined constant as the M_s in (14), we get

$$\mathcal{E}_v = \frac{8k^2 e^2 Z_{e0}^2}{3} \left[\frac{1 + \cos(2kM_v r_0)}{M_v r_0} - 2k \int_{M_v r_0}^{\infty} \frac{\sin 2kr}{r} dr \right] \quad (24)$$

Since r_0 and $M_v r_0$ are comparatively small, we may use the approximations

$$\cos(2kM_v r_0) \approx 1 \quad \int_{M_v r_0}^{\infty} \frac{\sin 2kr}{r} dr \approx \frac{\pi}{2} \quad (25)$$

then we have

$$\mathcal{E}_v = \frac{8k^2 e^2 Z_{e0}^2}{3} \left(\frac{2}{M_v r_0} - k\pi \right) \quad (26)$$

The acceleration dependent energy \mathcal{E}_a may be calculated from the acceleration dependent electromagnetic fields $E_{a\theta}$ and $H_{a\phi}$ in θ and ϕ directions. According to (2) and (3), \mathcal{E}_a is

$$\begin{aligned} \mathcal{E}_a &= \frac{1}{8\pi} \int (E_{a\theta}^2 + H_{a\phi}^2) dv \\ &= \frac{4k^4 e^2 Z_{e0}^2}{8\pi} \int_{r_{min}}^{r_{max}} \frac{2 \sin^2 \theta}{r^2} \cos^2 kr 2\pi r^2 dr \sin \theta d\theta \\ &= \frac{4k^4 e^2 Z_{e0}^2}{3} \left(r + \frac{\sin 2kr}{2k} \right) \Big|_{r_{min}}^{r_{max}} \end{aligned} \quad (27)$$

where $k = 2\pi/\lambda$, λ is wave length of the standing wave. The acceleration dependent energy \mathcal{E}_a contains a number of energy bands which are standing wave spherical shells with equal energy. We take the number of the bands each with width $\lambda/2$ as N . The central band is bisected by the electron at its center, half of its width is $\frac{1}{2} \cdot \frac{\lambda}{2}$, thus the upper limit is $r_{max} = (N + \frac{1}{2}) \frac{\lambda}{2}$. The lower limit here may be taken as $r_{min} = 0$, since $M_s r_0$ is small and here r_{min} is not in the denominator as above. Thus

$$\begin{aligned} \mathcal{E}_a &= \frac{4k^4 e^2 Z_{e0}^2}{3} \left(r + \frac{\sin 2kr}{2k} \right) \Big|_{r_{min}=0}^{r_{max}=(N+\frac{1}{2})\frac{\lambda}{2}} \\ &= \frac{4k^4 e^2 Z_{e0}^2}{3} (N + \frac{1}{2}) \frac{\lambda}{2} \\ &= \frac{4k^3 e^2 Z_{e0}^2}{3} (N + \frac{1}{2}) \pi \end{aligned} \quad (28)$$

The electric and magnetic fields of \mathcal{E}_v or \mathcal{E}_a respectively have equal time phase and they can not exchange energy within \mathcal{E}_v or \mathcal{E}_a alone, but the electromagnetic fields of \mathcal{E}_v and \mathcal{E}_a are out of phase $\pi/2$ in time, thus \mathcal{E}_v and \mathcal{E}_a can exchange their stored energy between each other. The exchange of energy between velocity and acceleration dependent fields of an oscillating electric dipole had been discussed by Mandel[12] and Booker[13] and they also showed that there was energy flow between these two fields. This kind of energy flow is also discussed in antenna theory. The condition for the complete energy exchange between \mathcal{E}_v and \mathcal{E}_a is

$$\mathcal{E}_v = \mathcal{E}_a \quad (29)$$

From (26) and (28), we get

$$\frac{8k^2e^2Z_{e0}^2}{3} \left[\frac{2}{M_v r_0} - k\pi \right] = \frac{4k^3e^2Z_{e0}^2}{3} \left(N + \frac{1}{2} \right) \pi \quad (30)$$

then we have

$$k_v = \frac{4}{M_v r_0 \left[\left(N + \frac{1}{2} \right) \pi + 2\pi \right]} \quad (31)$$

here k is labeled as k_v . This is a selection condition of k_v with M_v and r_0 . For the free electron to have any kind of persisted natural self-oscillation, both conditions of k_s in (22) and k_v in (31) should be satisfied at the same time, that is

$$k_s = k_v \quad (32)$$

which is the combined relation of possible natural self-oscillation.

5. Short discussion

Natural self-oscillation is popular in electric circuits, microwave structure or mechanical system. Standing wave solutions of electromagnetic waves are also used in antenna theory. Above energy analyses show that a free electron may also possess natural self-oscillation through the interaction with its own electromagnetic fields under certain conditions. These conditions are determined by the electron's standing wave modes, the number N of its standing wave bands in acceleration dependent fields and two numerical constants in its energy integrations. For an electron oscillating in a Penning trap as that in the experiment of Dehmelt et al for the “mono-electron oscillator”, there

is a strong static magnetic field along the oscillating direction of the electron to keep the oscillating electron localized and in stable movement. For the self-oscillation of a free electron this kind of external auxiliary is unnecessary, since localization is not a problem for a free electron. The natural self-oscillation will have some effects on the interaction of the electron with its environment and thus could be measured by suitable experiments. We will discuss these in connection with the stability of natural self-oscillation of a free electron. Here we only give an energy analysis and suggest that its existence is possible.

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